A Panel Method for Arbitrary Moving Boundaries Problems

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An integral representation of the velocity potential is derived for aerodynamic purposes which aims at handling complex geometry in complicated motion. A ground-fixed frame is used to describe the governing equation such that the Green's function is in its simplest form in this frame. The generalized derivatives are then introduced to allow the differentiations of discontinuous function across the moving boundary. Techniques to derive the integral formulas are originated from Farassat's approach in aeroacoustics for treating arbitrary moving-boundaries problems. In the present paper, however, integral formulas are expressed as combinations of a surface source operator and its derivatives and provide a unified method to represent both the scalar wave equation and the Ffowcs William-Hawkings equation. A panel method is then implemented based on the resulting formula and applied to rotating bodies, propeller blades, and a helicopter blade in forward flight. The numerical results are found to be in good agreement with experimental data and Long's results using Farassat's theory.

Nomenclature

A	= acceleration of the surface point
c	= speed of sound
dS	= differential area of surface $f=0$
$f(x,t)=0$	= description of body surface; $f>0$ outside and $f<0$ inside the surface
g	$=\tau - t + r/c$
M	$= \|v\|/c$; Mach number
M_n	$= v \cdot n/c$; Mach number in the normal direction
M_r	$= v \cdot \bar{r}/c$; Mach number in the $\bar{r}$ direction
n	$= \nabla f / \|\nabla f\|$, local outward unit normal on body $f=0$
p	= pressure field
R	$= \|1 - M_r\| r$
r	= vector $x - y$
r	$= \|x - y\|$; distance of source and observer
$\bar{r}$	= unit vector in r direction
v	= velocity of boundary point
x,t	= observer position, observer time
y,τ	= source position, source time
$\delta(f),\delta(g)$	= Dirac delta function
ρ_0	= density of undisturbed fluid
τ_i	= ith root of $g(\tau)=0$ for given (x,t); retarded time
ϕ	= velocity potential
$\dot{\phi}$	= time derivative of ϕ with respect to surface moving point
Φ	= surface source operator
ω	= angular velocity of the moving body at the surface point
$\bar{\partial}/\partial t, \bar{\nabla}, \ldots$	= bar over the differential operator denotes generalized derivatives
$\square^2$	= wave operator

I. Introduction

ALTHOUGH remarkable advances have been made in flowfield calculations using finite-difference and finite-element methods, the surface integral approaches of panel methods still offer distinct advantages for the subsonic and supersonic analysis in aerodynamics. In particular, panel methods offer greater versatility for practical applications to complicated configurations and are considerably more efficient in terms of computing effort.

However, as a result of the use of the Prandtl-Glauert equation as the governing equation in classical aerodynamics, the traditional panel methods[1-10] are limited to steady or constant speed problems and thus cannot be applied to the following problems.

Accelerated Moving-Boundary Problems

Since the exact Green's function is not available using the conventional aircraft-fixed frame if the aircraft itself is in accelerated motion, conventional panel methods are limited to steady rectilinear motions. Problems that cannot be solved using classical panel method[1-10] include a high-speed aircraft in roll, a high-speed helicopter in advanced motion, and the missile in accelerated motion.

Relatively Moving-Bodies Problems

The use of aircraft fixed-frame techniques implies the bodies considered must move without relative motion. This limitation in classical panel methods forced one to neglect the relative motion between bodies such as that of aircraft and separated store in the store separation problem.

A distinguished work that generalizes the panel method in aerodynamics for arbitrary moving problems is due to Morino and co-workers.[11-14] The works of Morino et al. originated from an integral formula of the wave equation with an arbitrary boundary that was derived in Ref. 11 and was intepreted as a generalization of the Huygens principle. Because the generalized formula was not explicitly expressed in terms of state (position, velocity, and acceleration) of the moving boundary, the integral equation must be transformed to a generalized Prandtl-Glauert frame (locally or globally) in practical computations that again resulted in an approximated equation neglecting the local acceleration of the boundary.

Another novel approach to panel methods, originated from solving the Ffowcs Williams-Hawkings (FW-H) equation for the pressure field to predict the aerodynamic noise of advanced high-speed propeller blades, has been developed by Farassat.[15,16] By using the ground-fixed frame instead of the aircraft-fixed frame and adopting the concept of generalized derivatives,[17] Farassat successfully derived several integral formulas for aeroacoustics. One of his aeroacoustic formulas for subsonic problems was extended to include the near-field problems by Long[18] and proved to be useful for calculating aerodynamic loading of rotating bodies. Another formulation

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valid for arbitrary motion was also extended to aerodynamic applications by Farassat himself.[20]

Although it is not necessary to consider the wake emitted from the trailing edge in the pressure formulations, the approaches of Farassat et al.[15,16,20] and Long[18,19] are not suitable for lifting problems due to the lack of lifting mechanism in the FW-H equation. This problem was resolved in a recent article by Long and Watts.[21] The concepts of the integral formulation of FW-H equation and the acceleration potential method were combined in this new approach. The result was an integro-differential equation that can be solved for the pressure loading. Similar to the acceleration potential method, the emanated wake was accounted for by an additional integral in their new method; thus, the lifting effect was included.

Except for their generality in considering arbitrary motions, the pressure formulations due to Farassat and Long and Watts are more convenient to obtain the inviscid loading than traditional panel methods using the velocity potential formulations because the pressure field is directly solved in the former formulations. The velocity formulations, however, have a merit over the pressure formulations when the velocity field is considered. The velocity field is obtained by differentiating the velocity potential field, which only requires instantaneous and local information about the velocity potential field. Obtaining the velocity field from the pressure field, however, involves a time integral of the gradient of the pressure field[21] and thus requires a complete knowledge of the time history of the pressure field. Since the velocity field is needed in considering the viscous-inviscid interaction problems, the velocity potential formulations are more economical than the pressure formulations in these cases.

In this paper, the integral formulation of the velocity potential for arbitrary moving boundary is derived parallel to Farassat's[15] and Long's[18,19] pressure formulations. The lifting effect in the present formulation is simulated by the doublet distribution on the wake, which is determined by the Kutta condition at the trailing edges of the wings.

Two important concepts in the aeroacoustics field are adopted in the present work. A ground-fixed frame is used to derive the integral formulas because the Green's function is independent of the motions of the boundary and remains in its simplest from in this frame. The generalized derivatives are then introduced to allow the differentiation of discontinuous functions across the moving boundary.

New integral formulas are derived in terms of surface source operators and their derivatives which enable a unified representation for both the scalar wave equation and the FW-H equation. A panel method is implemented based on the present formulation for arbitrary moving problems in aerodynamics. Numerical results for rotating bodies, propeller blades, and lifting and nonlifting helicopter blades are included and compared with experimental data and Long's[18] results.

In Sec. II, the integral representation of the scalar wave equation is derived in terms of a surface source operator and its derivatives. The corresponding representation for the FW-H equation is also given. A computational method based on the present theoretical result is described in Sec. III. Problems and difficulties encountered in the implementation of the panel method are also discussed. In Sec. IV, comparison of numerical results and the experimental data is made for rotating bodies and lifting and nonlifting propeller blades. Concluding remarks are made in Sec. V. Formulas for the time and space derivatives of the retarded time are listed in the Appendix.

II. Integral Formulas

Surface Source Operator

In a ground-fixed frame, the velocity potential in governed by the homogeneous scalar wave equation

$$\Box^2\phi = \nabla^2\phi - \frac{1\partial^2}{c^2\partial t^2}\phi = 0 \qquad \text{if} \qquad f(\boldsymbol{x},t)>0 \tag{1}$$

and the boundary condition

$$\nabla\phi\cdot\boldsymbol{n} = \boldsymbol{v}\cdot\boldsymbol{n} = cM_n \qquad \text{if} \qquad f(\boldsymbol{x},t)=0 \tag{2}$$

where $f = 0$ describes the moving boundary and $\boldsymbol{v}$ is the moving speed of the boundary point.

The definition domain of the function ϕ in Eq. (1) can be extended to the whole R^4 space by setting $\phi(\boldsymbol{x},t)=0$ for $f(\boldsymbol{x},t)<0$, which automatically satisfies Eq. (1) in the extended region. The definition of classical derivatives, however, must then be modified to allow one to take differentiation of discontinuous function across the surface $f(\boldsymbol{x},t)=0$. According to Farassat[17] or Jager[23], the generalized derivatives (or derivatives in the distribution sense) can be written as

$$\bar{\nabla}\phi = \nabla\phi + \phi\delta(f)\nabla f \tag{3a}$$

$$\frac{\bar{\partial}\phi}{\partial t} = \frac{\partial\phi}{\partial t} + \phi\delta(f)\frac{\partial f}{\partial t} \tag{3b}$$

The delta function terms in Eq. (3) can be explained as a modification of the classical derivatives to account for the finite jump across the discontinuous surface.

Substituting Eq. (3) into Eq. (1), one obtains

$$\bar{\Box}^2\phi = \bar{\nabla}^2\phi - \frac{1}{c^2}\frac{\bar{\partial}^2}{\partial t^2}\phi = \left(\nabla\phi\cdot\boldsymbol{n} - \frac{1}{c^2}\frac{\partial\phi}{\partial t}\frac{\partial f}{\partial t}\right)\delta(f)$$
$$+\ \bar{\nabla}\cdot\Big(\phi\nabla f\delta(f)\Big) - \frac{1}{c}\frac{\bar{\partial}}{\partial t}\left(\frac{\partial f}{\partial t}\phi\delta(f)\right) \tag{4}$$

which is the distribution equation for the velocity potential.

For a material moving surface described by $f(\boldsymbol{x},t)=0$, the following kinematic condition must also be satisfied:

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla f = \frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{n}\,|\nabla f| = 0 \tag{5}$$

Equation (4) can thus be written as

$$\bar{\Box}^2\phi = \left(\nabla\phi\cdot\boldsymbol{n} + \frac{1}{c}\frac{\partial\phi}{\partial t}M_n\right)|\nabla f|\delta(f)$$
$$+\ \bar{\nabla}\cdot\Big(\phi\boldsymbol{n}\,|\nabla f|\delta(f)\Big) + \frac{1}{c}\frac{\bar{\partial}}{\partial t}\Big(\phi M_n\,|\nabla f|\delta(f)\Big) \tag{6}$$

From Ref. 22, the formal solution of Eq. (6) can be expressed as

$$\phi(\boldsymbol{x},t) = \Phi\left(\phi_n + M_n\frac{1}{c}\frac{\partial\phi}{\partial\tau}\right) + \nabla\cdot\Phi(\boldsymbol{n}\phi) + \frac{\partial}{\partial t}\Phi(\phi M_n/c) \tag{7}$$

where the surface source operator Φ is defined as

$$\Phi[\sigma] = -\frac{1}{4\pi}\int\sigma(y,\tau)\,|\nabla f|\delta(f)\frac{\delta(g)}{r}\,\mathrm{d}y\,\mathrm{d}\tau \tag{8}$$

which takes the surface intensities function σ as an input and its output is a function of $(\boldsymbol{x},t)$.

The second and third terms on the right-hand side of Eq. (7) are combined to define the surface doublet operator with surface intensity function ϕ as an input. Equation (7) thus expresses the velocity potential at an arbitrary field point $(\boldsymbol{x},t)$ as the combination of surface source and doublet distributions with intensities

$$\phi_n + M_n\frac{1}{c}\frac{\partial\phi}{\partial\tau} \qquad \text{and} \qquad \phi$$

respectively.

In Eq. (8), the Dirac delta functions $\delta(f)$ and $\delta(g)$ can be eliminated by the following relation[22]:

$$|\nabla f|\delta(f)\delta(g)\,dy\,d\tau = \frac{dS}{|1-M_r|} \tag{9}$$

and one obtains

$$\Phi[\sigma] = -\frac{1}{4\pi}\int \sum_i \left(\sigma(y(\tau_i),\tau_i)\frac{1}{|1-M_r|r}\right)\Big|_{\tau=\tau_i} dS \tag{10}$$

where τ_i, the retarded time, is the ith root of $g(\tau) = \tau - t + r[y(\tau),x]/c = 0$, $y(\tau)$ is the trajectory of a surface point, and dS is the differential area of the moving surface $f = 0$.

Derivatives of Surface Source Distribution

The derivatives of surface source distribution with respect to time and space variables, Φ_t and $\nabla\Phi$, are defined as

$$4\pi\Phi_t[\sigma] = -\sum_i\int \frac{\partial}{\partial t}\left\{\sigma(y(\tau),\tau)\frac{1}{R}\right\}\Big|_{\tau=\tau_i} dS$$

$$= -\sum_i\int\left(\sigma\frac{\partial}{\partial t}\left(\frac{1}{R}\right) + \frac{1}{R}\frac{\partial\sigma}{\partial t}\right)\Big|_{\tau=\tau_i} dS$$

$$= -\sum_i\int\left(\sigma\frac{\partial}{\partial\tau}\left(\frac{1}{R}\right) + \frac{\dot\sigma}{R}\right)\frac{\partial\tau_i}{\partial t}\Big|_{\tau=\tau_i} dS \tag{11}$$

and

$$4\pi\nabla\Phi[\sigma] = -\sum_i\int \nabla_x\left(\sigma(y(\tau),\tau)\frac{1}{R}\right)\Big|_{\tau=\tau_i} dS$$

$$= -\sum_i\int\left(\sigma\nabla_x\left(\frac{1}{R}\right) + \frac{1}{R}\nabla_x\sigma\right)\Big|_{\tau=\tau_i} dS \tag{12}$$

Noting that $R = |1-M_r|r = R(y(\tau),x,\tau(y,x,t))$, we have

$$4\pi\nabla\Phi[\sigma] = -\sum_i\int\left\{\sigma\nabla_x\left(\frac{1}{R}\right) + \left[\sigma\frac{\partial}{\partial\tau}\left(\frac{1}{R}\right) + \frac{\dot\sigma}{R}\right]\nabla_x\tau\right\}\Big|_{\tau=\tau_i} dS \tag{13}$$

The derivations of the derivatives

$$\frac{\partial\tau}{\partial t},\quad \nabla_x\tau,\quad \frac{\partial}{\partial\tau}\left(\frac{1}{R}\right),\quad \text{and}\quad \nabla_x\left(\frac{1}{R}\right)$$

are listed in the Appendix.

After inserting Eqs. (A.3), (A.4), (A.5), and (A.6) into Eqs. (11) and (13), one has

$$4\pi\Phi_t[\sigma](x,t) = -\sum_i\int\frac{c}{r(1-M_r)|1-M_r|} \times\left(\sigma\frac{A\cdot r/c^2 + (M_r - M^2)}{r(1-M_r)} + \dot\sigma/c\right)\Big|_{\tau=\tau_i} dS \tag{14}$$

$$4\pi\nabla\Phi[\sigma](x,t) = -\sum_i\int\frac{1}{r(1-M_r)|1-M_r|} \times\left\{\sigma\frac{M-\bar r}{r} - \bar r\left[\sigma\frac{A\cdot r/c^2 + (M_r - M^2)}{r(1-M_r)} + \dot\sigma/c\right]\right\}\Big|_{\tau=\tau_i} dS \tag{15}$$

Substituting Eqs. (14) and (15) into Eq. (7), one finally obtains the integral representation for the solution of the scalar wave equation:

$$4\pi\phi(x,t) = -\sum_i\int\left(\nabla\phi\cdot n + \frac{1}{c}\frac{\partial\phi}{\partial\tau}M_n\right)\frac{1}{r|1-M_r|}\Big|_{\tau=\tau_i} dS$$

$$-\sum_i\int\frac{1}{r(1-M_r)|1-M_r|}\left\{\phi(M_n - \bar r\cdot n)\frac{A\cdot r/c^2 + (1-M^2)}{r(1-M_r)}\right.$$

$$\left. + (M_n - \bar r\cdot n)\dot\phi/c + \phi[A/c^2 + (M-\bar r)\times\omega/c]\cdot n\right\}\Big|_{\tau=\tau_i} dS \tag{16}$$

Equation (16) is the main result of the present work.

With the present terminologies, the solution of the FW-H equation[18] can also be expressed as

$$4\pi p(x,t) = 4\pi\frac{\partial}{\partial t}\Phi[-\rho_0 cM_n] + 4\pi\nabla\cdot\Phi[np]$$

$$= \sum_i\int\frac{c^2}{r(1-M_r)|1-M_r|}\left(\rho_0 M_n\frac{A\cdot r/c^2 + (M_r - M^2)}{r(1-M_r)}\right.$$

$$\left. + \rho_0(A + v\times\omega)\cdot n/c^2\right)\Big|_{\tau=\tau_i} dS$$

$$-\sum_i\int\frac{1}{r(1-M_r)|1-M_r|}\left\{(M_n - \bar r\cdot n)\frac{p}{r}\right.$$

$$-\left[p\bar r\cdot n\frac{A\cdot r/c^2 + (M_r - M^2)}{r(1-M_r)}\right.$$

$$\left.\left. + \bar r\cdot(\dot p n + (\omega\times n)p)/c\right]\right\}\Big|_{\tau=\tau_i} dS \tag{17}$$

which is, in fact, consistent with Long's formulation[18] after some manipulations.

III. Computational Method

It is noticed that the time derivative terms in the ground-fixed and aircraft-fixed frame are related by

$$\frac{\partial\phi}{\partial\tau} = \dot\phi - v\cdot\nabla\phi \tag{18}$$

The gradient operator can also be decomposed as

$$\nabla\phi = \phi_n n + \nabla_2\phi = cM_n n + \nabla_2\phi \tag{19}$$

at the moving boundary, where ∇_2 represents a two-dimensional gradient operator on the plane tangent to the moving boundary, and $\nabla_2\phi$ thus represent the tangent components of the velocity field.

Substituting Eqs. (18) and (19) into Eq. (7), it follows that

$$\phi(x,t) - \Phi[M_n(\dot\phi/c - M\cdot\nabla_2\phi)] - \Phi_t[\phi M_n/c] - \nabla\cdot\Phi[n\phi]$$

$$= \Phi[(1-M_n^2)cM_n] \tag{20}$$

Now if the time derivative terms in Eq. (20) are neglected (which is adequate under the quasisteady assumption) and if the observer position x approaches the moving boundary at time t, Eq. (20) becomes a boundary integral equation for the surface intensity of ϕ once the motion of the boundary is given.

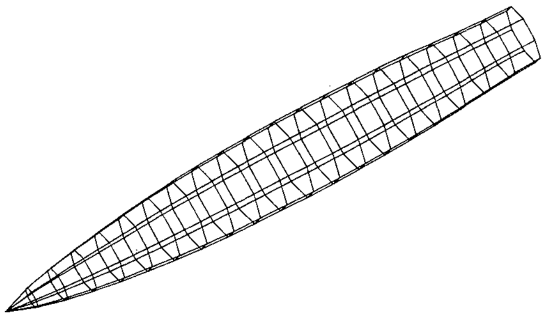

Fig. 1 Haack-Adams body.

To solve Eqs. (20) or (16) numerically, the moving boundaries are first approximated by a finite number of small flat elements (panels). The surface source and doublet intensities are assumed to be constant within each element and are represented by their values at the center point of the element. The 2D tangent velocity term $\nabla_2\phi$ in each element can then be obtained by using the finite difference of ϕ and can be numerically expressed as a linear combination of ϕ at the neighborhood elements of the element considered. The resulting equation is then applied to the center point of each element to yield a system of algebraic equations for the surface doublet intensity at each element. Once solved, the velocity of the fluid at the moving boundary is obtained from Eq. (19). Pressure distribution on the boundary can thus be obtained from the velocity distribution by Bernoulli's equation.

The present method solves the velocity potential directly from the integral equation. These types of formulations are found to be insensitive to the position of the control point (observer), which can also be demonstrated using similar formulations such as those given in Ref. 5.

The algorithm used to compute the integrals in Eqs. (16) and (17) is essentially the same as the used in Long's[19] method. The integrals in Eqs. (16) and (17) become singular as the point x approaches one of the elements considered. Long[18,19] has derived the contribution of singular integral for the FW-H equation, Eq. (17), such that it can be used as an aerodynamic equation. A corresponding derivation of contribution of singular integrals for Eq. (16) was also discussed by Lee[24] and was found to be of value $2\pi\phi$. For the point x not lying in the element, the integrals in Eq. (16) are regular and thus are calculated numerically by Legendre-Gaussian quadrature with 4–64 node points according to the distance from x to the center of element.

Another type of singularity, the Doppler singularity, must be mentioned for supersonically moving problems. The Doppler singularity arises when the Doppler factor, $1-M_r$, becomes zero, which can occur only for supersonic problems. In such a situation, the integrals in Eqs. (16) and (17) become divergent; thus, the improper integral "finite part" or some other regularization process must be employed. However, the integrals in Eqs. (16) and (17) can be calculated in the classical sense even in supersonically moving problems if the Doppler singularity does not occur within the element considered.

In the present implementation, only subsonically moving problems are considered, and the numerical results are discussed in the following section.

IV. Numerical Results

In this section we present numerical results for several different types of bodies in various combinations of motion. These different types of motions were used to illustrate the generality of the method.

Body of Revolution

A Haack-Adams body, as depicted in Fig. 1. moving with $M=0.8$, was calculated by the present method. The Haack-Adams body was approximated by 20×8 panels. The calculated result is shown in Fig. 2 and is in good agreement with the experimental data.[25]

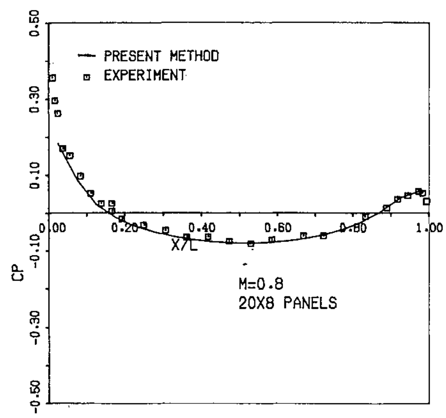

Fig. 2 Comparison of computed and measured pressure distribution of Haack-Adams body moving with $M = 0.8$.

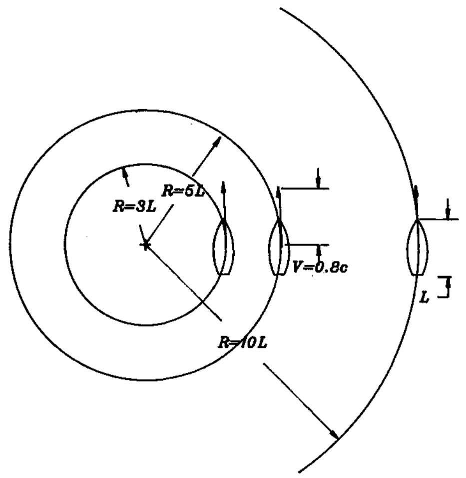

Fig. 3 Haack-Adams bodies rotate about a fixed axis.

Rotating Body

In this case, pressure distributions of the Haack-Adams body rotated around a fixed axis with constant radius $R = 3L$, $R = 5L$, and $R = 10L$, respectively, were calculated, where L is the length of the Haack-Adams body. The angular velocities were adjusted such that the moving velocities of the center point of the rotating bodies remained a constant value of $0.8c$, as shown in Fig. 3. The pressure distributions corresponding to $R = 3L$ and $10L$ cases are shown in Fig. 4. The pressure changes from a nonuniform distribution in $R=3L$ case to a more uniform distribution in $R = 10L$ case as expected because the pressure distribution tends to become that due to rectilinear motion when the rotating radius is going to a large value. The pressure distributions along the meridian line at $\theta = 22.5$ deg for the $R = 3L$, $5L$, and $10L$ cases are plotted in Fig. 5. Again, the pressure distribution corresponding to the largest value of radius length ($R = 10L$ case) is in better agreement with the experimental data obtained from the body moving with rectilinear velocity of $0.8c$.

Nonlifting Helicopter Rotor Blade

The next example was performed for a rotating NACA 0012 blade. The results were compared with the experiments of Gray et al.[26] and numerical data of Long.[18] The blade was approximated by $12 \times 9 \times 2$ panels. Tip Mach number was 0.25, and angle of attack was zero.

In Fig. 6, the variations in the pressure coefficient along the chord are shown for three spanwise locations: 94, 98, and 99.5%. Experimental data[26] and numerical results by Long[18,19] and the present method are compared. All of the data are in good agreement with each other except near the leading edge, where it is found that solutions by the scalar wave equation seem to agree better with the experimental data than those by the FW-H equation.

The program is developed on a Convex C-1 mini/supercomputer, and the source program is compiled without the optimization and vectorization options. The required CPU time is 840 s for this case and about 1200 s for the case using $15 \times 9 \times 2$ panels.

Lifting Helicopter Rotor Blade

Figure 7 presents similar results for the blade at an angle of attack of 6.18 deg. To account for the lifting effect, wake emitted from the sharp trailing edge must also be modeled with doublet panels. Doublet intensity on the wake is determined by the Kutta condition in the same form as that used by Morino et al.[5] and by Magnus and Epton,[10] which requires a constant doublet distribution along the line emitted from the same edge point. This type of Kutta condition is valid for the present steady problem, but the shape of the wake is modeled only by a plane surface rather than by a more appropriate helical one.

As indicated in Fig. 7, the numerical result agrees well with experimental data by this simple wake model. This same case was also calculated by Long[18] solving the FW-H equation, and his result is better than ours, especially on the upper surface, which may be explained as due to our use of a simple wake model. In his method, however, the lifting effect was considered by replacing the right-hand side of the system of the algebraic equations (for the panels on the upper and lower surfaces at the trailing edge) by the average of the two values. This must be done for all the panels from the trailing edge to the thickest part of the cross section. However, this method cannot be directly extended to more general wing configurations such as the cases for unsymmetrical or cambered cross section. This shortcoming was also indicated in a recent work

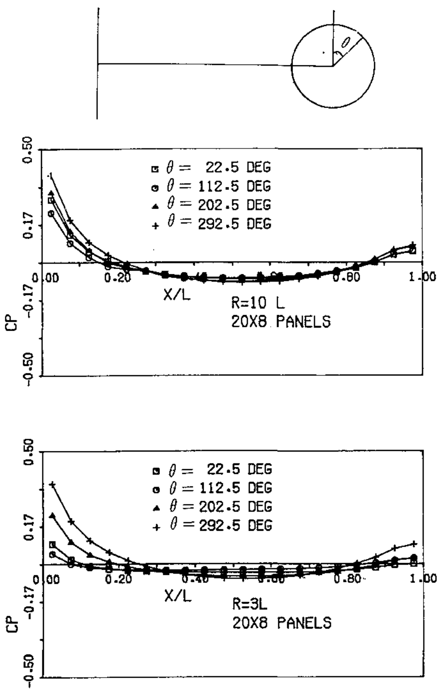

Fig. 4 Calculated pressure distribution for Haack-Adams bodies rotate with 10*L* and 3*L* radius.

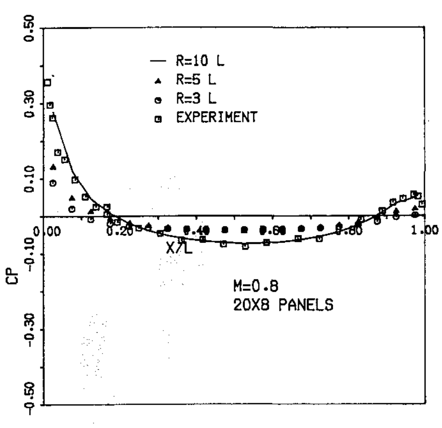

Fig. 5 Pressure distribution along θ = 22.5 deg of Haack-Adams bodies rotate with 3*L*, 5*L*, and 10*L*.

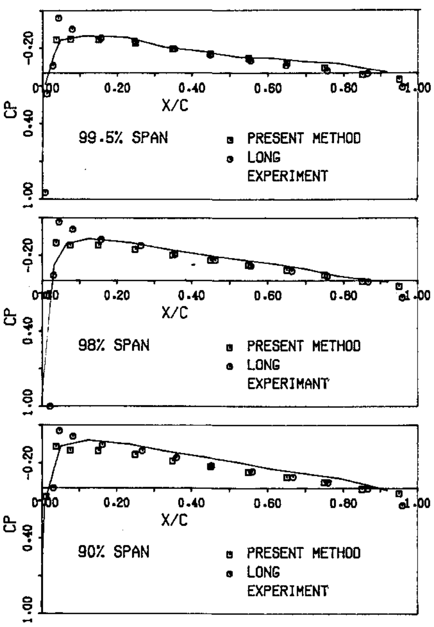

Fig. 6 Pressure distribution of NACA 0012 blade at three-tip sections.

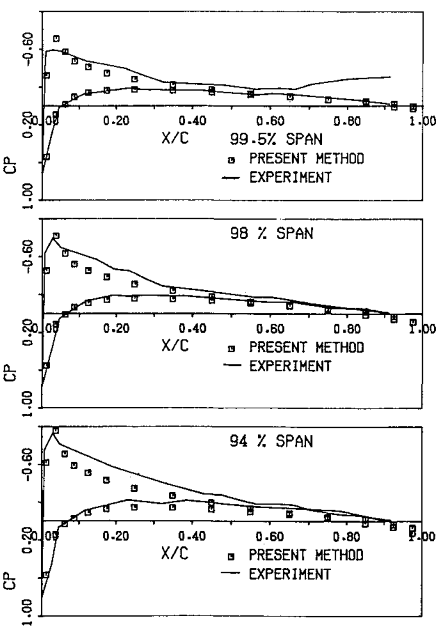

Fig. 7 Pressure distribution of NACA 0012 blade at three-tip sections, angle of attack = 6.18 deg.

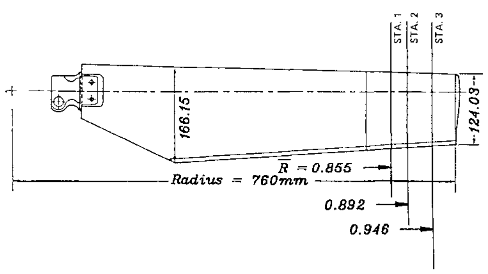

Fig. 8 Planform geometry of an ONERA nonlifting model rotor blade with straight tip.

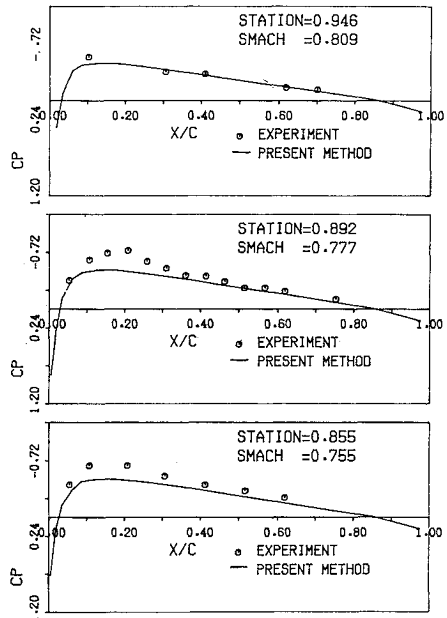

Fig. 9 Comparison of computed and measured surface pressure for a nonlifting blade in forward flight.

by Long and Watts[21] and was the motivation for their newly developed method based on the FW-H equation.

Helicopter Rotor Blade in Forward Flight

The ONERA rotor blade is considered in this example, as shown in Fig. 8. The blade was tested with tip Mach number 0.6 and advanced ratio 0.4. The calculations were carried out at an azimuth angle of 90 deg under quasisteady assumption, although the realistic problem is an unsteady one. The numerical results at 0.946, 0.892, and 0.855 spanwise location with section Mach number SMACH of 0.809, 0.777, and 0.755, respectively, are shown in Fig. 9. Experiments[27] indicate that the small perturbation assumption is not adequate in this case become of the existence of mixed subsonic-supersonic flowfields in this problem. However, the numerical results provide a qualitative description in this nonlinear case.

V. Conclusions

In this paper, an integral formula for linearized compressible potential flow is derived based on the concepts of the generalized derivatives and the ground-fixed frame. Different from the integral equation based on the Prandtl-Glauert equation, the present formulation expresses the integral in terms of the states (position, velocity, and acceleration) of the boundary point at the retarded times. The resulting formula has no restrictions on the geometry, type of motion , and moving speed of the boundary and thus provides a unified theoretical background for the analysis of aerodynamics due to complex geometry in complicated motion. A panel method is then implemented based on the derived formulation for bodies moving in a combination of rectilinear and rotary motions.

Numerical results for rotating bodies, lifting and nonlifting rotor blades, and rotor blade in forward flight are given and compared with experimental data. Good agreement between numerical results and experimental data are found, which demonstrates the applicability of the present approach.

However, for unsteady problems, formulas derived in the present work are valid only for solid boundaries. The contribution due to the unsteady motion of the wake is not considered in the present work and is a topic under research.

To apply the method to supersonic problems, however, regularization of the divergent integral due to the effect of Doppler factor must first be carried out either numerically or theoretically, which is not discussed in the present paper. This type of singularity can be treated by expressing the integral equation in the local Galilean coordinate system, which results an equation similar to the integral equation for the Prandtl-Glauert equation. Regularization of the divergent integral can then be performed by calculating the integral in the sense of Hadamard's finite part. Details of the local Galilean coordinate method will be reported in a separate article.

Appendix

Derivatives of Retarded Time, $\partial\tau/\partial t$ and $\nabla_x\tau$

The derivatives $\partial/\partial t$ and $\nabla_x\tau$ in Eqs. (11) and (13) can be obtained by taking differentiation of the retarded time equation, $g=\tau-t+r/c=0$, as follows:

$$\left(1+\frac{1}{c}\nabla_y r\cdot\frac{\partial y}{\partial\tau}\right)\frac{\partial}{\partial t}\tau=1 \tag{A.1}$$

and

$$\left(1+\frac{1}{c}\nabla_y r\cdot\frac{\partial y}{\partial\tau}\right)\nabla_x\tau=-\frac{1}{c}\nabla_x r \tag{A.2}$$

Let $r=x(t)-y(\tau)$, $r=|r|$, and $\bar{r}=r/r$. Noting that $\partial y/\partial\tau=v$ is the velocity of the source point, Eqs. (14) and (15) become

$$\frac{\partial\tau}{\partial t}=\frac{1}{1-M_r} \tag{A.3}$$

$$\nabla_x\tau=\frac{-\bar{r}/c}{1-M_r} \tag{A.4}$$

Derivatives $\partial/\partial\tau(1/R)$ and $\nabla_x(1/R)$

The derivatives $\partial/\partial\tau(1/R)$ and $\nabla_x(1/R)$ in Eqs. (11) and (13) can be obtained by the following:

$$\frac{\partial}{\partial\tau}\left(\frac{1}{R}\right)=\frac{\partial}{\partial\tau}\left(\frac{1}{|1-M_r|r}\right)=\frac{\partial}{\partial\tau}\left(\frac{1}{|r-v\cdot r/c|}\right)$$

$$=\frac{A\cdot r/c+c(M_r-M^2)}{r^2(1-M_r)|1-M_r|} \tag{A.5}$$

$$\nabla_x\left(\frac{1}{R}\right)=\nabla_x\left(\frac{1}{|1-M_r|r}\right)=\nabla_x\left(\frac{1}{|r-v\cdot r/c|}\right)$$

$$=\frac{M-\bar{r}}{r^2(1-M_r)|1-M_r|} \tag{A.6}$$

where A is acceleration of the surface point.

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